MULTIFRACTAL ANALYSIS OF U.S. INDUSTRIAL PRODUCTION OVER THE BUSINESS CYCLE 1919-2022

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Resumen: Se analiza el índice estadounidense de producción industrial para detectar la presencia de memoria larga durante el período 1919–2022. Se calculan exponentes de Hurst en ventanas de veinticuatro meses anteriores, centradas en y posteriores al inicio de cada recesión, con el fin de examinar la evolución de la producción. Este enfoque permite asimismo explorar si la actividad económica está impulsada por la innovación empresarial (memoria larga antipersistente con $0 \leq H < ½$) o por efectos Cantillon insostenibles derivados de la expansión monetaria y del comportamiento gregario (memoria larga persistente con $½ < H \leq 1$). El carácter multifractal de la producción industrial varía de manera sistemática a lo largo de las recesiones y expansiones. Los últimos meses de una expansión insostenible se caracterizan por la supresión de la antipersistencia empresarial normal del mercado y la imposición de una memoria larga fractal. La fase de recuperación restablece la antipersistencia asociada con la experimentación empresarial y elimina los procesos de memoria larga. El punto de inflexión que marca el inicio de una recesión también reinicia el carácter fractal de la producción industrial.

Palabras clave: Ciclo económico; exponente de Hurst; estructura de producción.

Clasificación JEL: B53; C22; C43.

Abstract: The U.S. index of industrial production is examined for long memory over 1919-2022. Hurst exponents are computed over twenty-four month windows preceding, centered on, and following the onset of each recession to examine how output evolves. This also enables us to explore whether economic activity is

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driven by entrepreneurial innovation (anti-persistent long memory with $0 \leq H < ½$) or by unsustainable Cantillon effects of monetary expansion and herding (persistent long memory with $½ < H \leq 1$). The multifractal character of industrial production varies systematically during recessions and expansions. The last months of an unsustainable expansion are characterized by suppression of the market's normal entrepreneurial anti-persistence and the imposition of fractal long memory. Recovery re-establishes the anti-persistence associated with entrepreneurial experimentation and wipes out long memory processes. The turning point marking the start of a recession also resets industrial production's fractal character.

Keywords: Business cycle; Hurst exponent; production structure.

JEL Classification: B53; C22; C43.

1. **Introduction**

The Hurst (1951) exponent distinguishes between stable and unsustainable processes. Hurst analysis has been used to address a variety of problems (Weron & Przybyłowicz 2000; Alvarez-Ramirez et al 2002; Katsev & L'Heureux 2003; Nourozzadeh & Jafari 2005; Li 2006; Cajueiro & Tabak 2007; Cannon et al 2007; Ellis 2007; Liu, DiMatteo, & Lux 2007; Mielniczuk & Wojdyłło 2007; Alvarez-Ramirez, Echevverria, & Rodriquez 2008; Movahed & Hermanis 2008; Barunik & Kristoufek 2010; Serinaldi 2010) including analysis of financial time series (Koppl 2002; Grech & Mazur 2004; Mulligan 2004, 2006, 2010, 2014; Mulligan & Lombardo 2004; Gu, Chen, & Zhou 2007; Alvarez-Ramirez et al 2008; Eom et al 2008; Grech & Pamuła 2008; Mulligan & Banerjee 2008, 2010; Costa, Galati, & Rognoni 2009; Kumar & Deo 2009; Romero et al 2010; Morales et al 2012), institutional evolution (Lee et al 1998), hydrological studies (Hurst 1951, Movahed & Hermanis 2008), ecological dispersion (Peng et al 2012), information science/cybersecurity studies (Chen & Papamarcou 1995; Li 2006), and civil engineering structures (Zheng & Fan 2001; Moyo & Brownjohn 2002; Gentile & Messina 2003; Melhem & Kim 2003; Nagarajalah & Varadarajan 2005; Pakrashi & Ghosh 2009; Pakrashi, O'Connor & Basu 2010; Arrigan et al 2011; Pakrashi et al 2013). Hurst analysis has also been used to examine extreme local volatility (Liu, DiMatteo, & Lux 2007; Grech & Pamuła 2008; Morales et al 2012). Most Hurst studies of financial data use daily-observed or even infra-daily series such as equity prices. Many of these time series, e.g., the S&P 500 index, exhibit some of the same behavior over the business cycle as more sparsely-sampled macroeconomic variables like gross domestic product (GDP) or the index of industrial production (IIP) examined here. Because it is observed monthly, the IIP is more amenable to fractal analysis techniques, particularly wavelet analysis, than quarterly-observed GDP.

Austrian business cycle (ABC) theory (Mises 1912, 1949; Hayek 1931, 1933, 1939, 1941; Garrison 2000) traces the cause of recessions to unsustainable resource allocation imposed by expansionary monetary policy. Money supply growth creates additional funds for investment but simultaneously depresses interest rates, relaxing saver-investor incentives to ration credit to higher-yielding activities. Less-productive, lower-yielding activities appear more competitive in this low-interest environment. These lower-yielding activities would not have been financed before monetary expansion, ultimately lowering productivity, return on investment, and long-term growth. Monetary expansion and lower interest rates also suppress saving and stimulate consumption spending.

Richard Cantillon (1680-1734) first identified the asymmetric expansion of production in sectors where new money first increases demand—hence this unsustainable reallocation of production, localized in particular industrial sectors and caused by monetary expansion, is known as a Cantillon effect (Cantillon 1755; Blaug 1977; Bordo 1983; Hülsmann 2002)[1]. We saw unsustainable expansions in new technology and telecommunications leading up to the 2001 recession, and in finance, construction, and real estate prior to the 2007-2009 Great Recession. Contemporary macroeconomics largely neglects this form of monetary non-neutrality and the unsustainability it introduces. Recessions occur when unsustainability becomes critical and triggers a contraction that reallocates productive resources back to sustainable uses. Often assets that were acquired at high cost during the expansion are written

[1] Blaug (1977) introduced the expression Cantillon effect (p. 21 of the 1996 5th edition).

down and liquidated at fire-sale prices during the recession, which aids in reallocating them to higher-yielding uses. Real savings were actually destroyed during the unsustainable expansion when firms acquired assets at unrealistically high cost during the expansion, but this does not become apparent until the correction occurs and asset prices are readjusted downward[2].

Industrial output results from decisions of many thousands of entrepreneurial planners. This suggests that much of the fractal character of underlying complex processes may be lost from aggregation, but this is clearly a case where the feedback among many individual allocative and entrepreneurial decisions are not deterministically related or stable over time. The extreme aggregation of the IIP makes frequent findings of anti-persistence even more remarkable.

The rest of the paper is organized as follows. Section 2 describes the data, section 3 describes the method for estimating the Hurst exponent, section 4 presents a digest of ABC theory, section 5 presents empirical results, section 6 presents concluding comments.

2. **Data**

The IIP for the U.S. (INDPRO) (Federal Reserve System 2023) is a seasonally-adjusted monthly index with a base year currently set at 2007 = 100, from the Federal Reserve monthly G.17 Statistical Release, Capacity Utilization and Industrial Production. The IIP measures output of U.S. manufacturing, mining, and electric/gas utility establishments from January 1919 to November 2022. The IIP is a measure of real or inflation-adjusted output. Since 1997, the

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[2] Standard accounting practices are biased toward overvaluing assets acquired in the late stages of an unsustainable expansion. Unsustainable expansion systematically overprices assets throughout the economy. These unsustainably high prices collapse during a market adjustment. Asset valuation bias is equally prevalent with U.S. Generally Accepted Accounting Principles (GAAP) published by the Financial Accounting Standards Board (FASB) and with International Financial Reporting Standards (IFRS) maintained by the IFRS Foundation and the International Accounting Standards Board (IASB) (Posner 2010). It will not be appreciably diminished under the convergence in standards anticipated by the Norwalk Agreement.

IIP has aggregated 312 North American Industrial Classification System (NAICS) output categories.

The IIP grows exponentially but its annualized percent change computed as

$$\%\Delta(IIP)_t = (1 + ((IIP_t - IIP_{t-1})/IIP_{t-1}))^{12} - 1.$$

is difference-stationary by construction. Data plots are shown in Figure 1. For comparison consider the exponentially-growing Fibonacci series: 1, 1, 2, 3, 5, 8, 13, 21, 34, …. Its first-difference 0, 0, 1, 1, 2, 3, 5, 8, 13, … also expands exponentially, reaching 2584 after 20 periods. This series is clearly not stationary. However the percent change, being continuously renormalized or rescaled, converges rapidly to 0.618034 after 20 intervals.

FIGURE 1.
DATA PLOTS OF IIP 1919-2022 (ANNUALIZED PERCENT CHANGE)

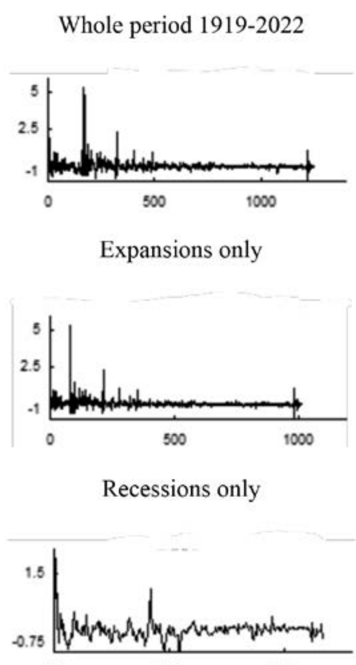


TABLE 1.
WHOLE PERIOD HURST EXPONENT ESTIMATES FOR THE INDEX OF INDUSTRIAL PRODUCTION 1919-2022, ESTIMATED BY VARIOUS METHODS AND FOR VARIOUS DATA TRANSFORMATIONS

Time series	*R/S*		*Power Spectral Density*			*Roughness-Length*		*Variogram*		*Wavelets*	
	$H_{R/S}$	*s.d.*	H_{SD}	*s.d.*	β	$H_{R\text{-}L}$	*s.d.*	H_V	*s.d.*	H_{Wz}	H_{Wt}
IIP	0.603	0.0843	0.582	1.055	2.056	0.705	0.0113	0.625	0.176	0.865	0.953
% Δ	0.250	0.0174	0.484	2.190	1.968	0.148	0.00126	0.008	0.177	0.082	0.054
ln(1st Δ)	0.249	0.0181	0.436	3.421	1.872	0.148	0.00129	0.008	0.178	0.082	0.059
Annualized $(1+\%\ \Delta)^{12}$*-1*	0.287	0.0064	0.321	8.335	1.641	0.182	0.00105	0.0156	0.157	0.056	0.065
*12*ln(1st Δ)*	0.249	0.0181	0.359	3.489	1.791	0.148	0.00128	0.008	0.178	0.082	0.059
Annualized $ln(1st\ \%\ \Delta)^{12}$	0.279	0.0079	0.487	5.583	1.935	0.174	0.00109	0.014	0.161	0.067	0.060

Notes:

1. With 1247 observations of the IIP from 1919 to 2022, 1246 after differencing, 1080 are used to estimate H by rescaled range (R/S), providing 22 different sample sizes with different average ranges to be rescaled.
2. Power Spectral Density H's are computed with time series transformed by successive random addition. Otherwise H = 0 for each representation of the rate of change of the IIP, though standard deviations and beta coefficients vary slightly. Power spectral density is unable to distinguish between blue, white, and pink noise when the slope β falls in the range ($-1 < \beta < 1$). In this case this is a result of transforming the time series to make it difference stationary. Thus the further successive random addition transformation is necessary.
3. H's are always identical for the log-first-difference and 12*ln(1st Δ) since they are linear transformations of each other. The apparent discrepancy in $H_{R\text{-}L}$ standard deviations is due to rounding.
4. Wavelet H's are computed using 1024 observations out of 1246 or 1247 total observations. Wavelet Hurst exponents are alternatively computed by zooming on the samples (H_{Wz}) and truncating the data (H_{Wt}).

A Dickey-Fuller (1979) unit root test on the raw IIP series yields a t-statistic of 1.01, failing to reject the null of a unit root. The same test on the annualized percent change yields a t-statistic of -19.62, rejecting the null of a unit root, i.e., indicating that the annualized percent change series is stationary. The a priori consideration that the percent change is stationary by design should govern regardless of the outcome of unit root tests since statistical tests reject the null hypothesis α percent of the time when the null is true.

Business cycle turning points (USREC) (NBER 2023) are taken from the National Bureau for Economic Research (NBER) based Recession Indicators for the United States from the Period following the Peak through the Trough.

Note that regardless of which method is used to compute the Hurst exponent, it is unambiguously > ½ for the non-stationary IIP in levels, and equally unambiguously < ½ for the difference-stationary % Δ IIP. There is no difference worth noting about the various ways of representing the percent growth in industrial production.

3. **Method**

The Hurst exponent was introduced for hydrological studies of the Nile (Hurst 1951). It measures the relationship between variable inflows and the reservoir necessary to guarantee a constant or constantly cyclical outflow. Inflow variability can range from completely random to constant or periodic. In macroeconomics we apply the Hurst exponent to assessing the sustainability of a time series—in this case with the IIP, we examine whether the output series grows sustainably or not. H measures persistent long memory, ranging from ½ for the normal or Gaussian distribution to 1 for the extremely fat-tailed Cauchy or Lorentz distribution is shown in Figure 3. However, many macroeconomic time series, far from displaying persistent long memory, display anti-persistent long memory or negative serial correlation with $0 \leq H < ½$ as shown in Figure 2.

Interpretation of the Hurst Exponent is given in Figure 2. Hurst analysis of industrial production enables us to distinguish between (a.) sustainable resource allocation driven by organic changes in supply and demand due to ordinary entrepreneurial experimentation (anti-persistent long memory where H < ½) (Peters 1999) or (b.) unsustainable Cantillon effects (Blaug 1977) due to monetary expansion and herding behavior (persistent long memory where H > ½) (Koppl 2002).

FIGURE 2.
INTERPRETATION OF HURST EXPONENT H

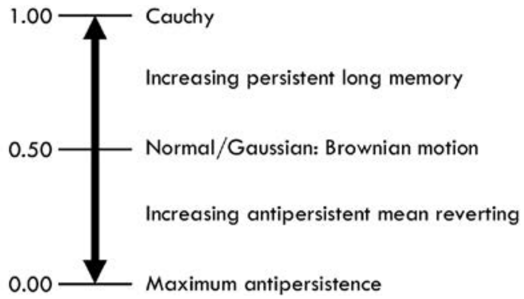
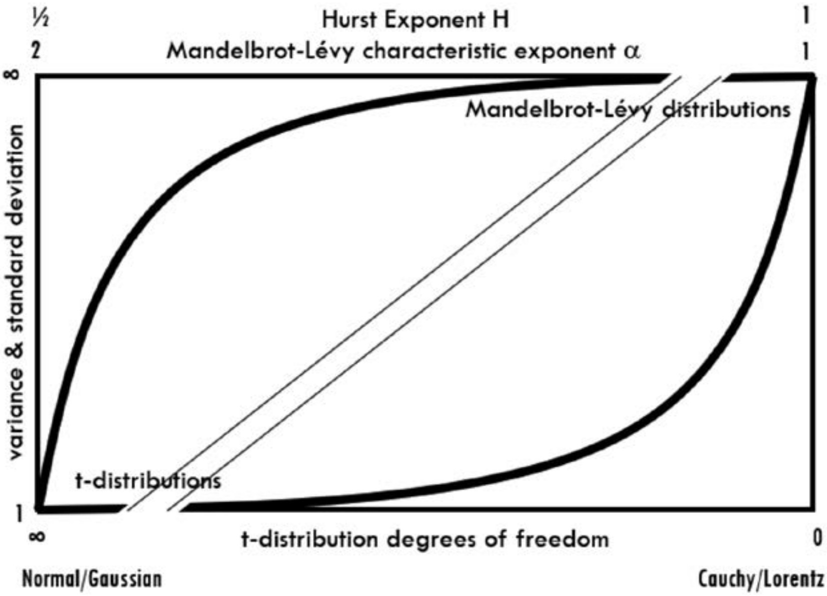


Figure 3 illustrates relationships for the long memory upper half of figure 2 where $\frac{1}{2} < H \leq 1$. Mulligan (2014) found that price adjustment is normally anti-persistent and dominated by entrepreneurial experimentation, but that in some sectors it becomes systematically biased where an exogenous driver like expansionary monetary policy renders price adjustment either more persistent or less anti-persistent. Mulligan (2017) examined real output and capacity utilization, finding that middle-stage capacity utilization was driven less by persistent external driving factors, compared with early and late stages of production. Both findings support ABC theory.

The conventional rescaled-range (R/S) method introduced by Mandelbrot (Mandelbrot & Wallis 1969; Mandelbrot 1972, 1975, 1977) requires at least 48 observations. Roughness-length, power spectrum, and variogram methods require a minimum of 120 observations. Fourier transforms used to estimate Hurst exponents by power spectral density have low power for sparsely-sampled data (Trzasko & Manduca 2009), but for a macroeconomic index like the IIP, all infra-monthly variations have been removed and longer-term movements can be used to examine longer-term phenomena such as the business cycle. The present paper computes Hurst exponents with wavelet transforms due to data limitations imposed by the number of months between business cycle peaks and troughs (Mulligan 2024). Hurst analysis can be complicated by irregular data sampling (Aldroubi 2002) but fortunately this does not apply to the index of industrial production, which is estimated for regular monthly intervals. A more binding constraint is the IIP's monthly frequency (Xu, Huang, & Zhang 2009). Although this is less of problem than for quarterly-observed GDP, the length of time series analyzed here, 24-48 months, approaches the lower limit (16 observations) for which Hurst exponents can be estimated by wavelets.

FIGURE 3.
RELATIONSHIPS AMONG LONG MEMORY (½ < H ≤ 1) DISTRIBUTIONS



Wavelet estimation of H is often superior to other methods (Simonsen, Hansen, & Nes 1998; Wu 2020), though potential small-sample bias must be kept in mind in interpreting results—as we will see below, a small number of estimated Hurst exponents are negative, which we interpret as zero. This suggests incompletely detrended series or very strong negative serial correlation. Wavelet analysis is the only fractal method that can be applied to monthly series over the course of a recession, though many recessions, which can be as brief in duration as two quarters, are too short to allow computation of H even by wavelets. Ten years of monthly data are provided by many business cycle expansions, but fortunately no recession has ever lasted that long. Wavelet Hurst exponent determination is comparable in accuracy to Fourier power spectral density analysis when larger samples are available. However, for short samples wavelets outperform the Fourier power spectrum method by a large margin (Simonsen, Hansen, & Nes 1998). Furthermore, the relatively brief sample ranges available over a given recession are too small to allow for other methods for determining H. Wavelet Hs were computed by zooming rather than truncation (Said & Perlman 1996; Atto et al 2016).

4. **ABC Theory**

Drawing on Bischoff's (1970) distinction between "putty" financial capital and less-flexible installed or physical "clay" capital, Mulligan (2014) noted that business cycle expansions result from an overaccumulation of low-yielding physical capital. Because physical capital is longer-lived than consumption goods, large accumulations of low-yielding capital tend to suppress growth and output. After a correction this capital has to be reallocated to alternative uses which are predominantly lower-yielding and less productive than originally anticipated. Only entrepreneurial planners' subjective expectations confer capital character on capital goods or the human capital embodied in worker experience, training, and education.

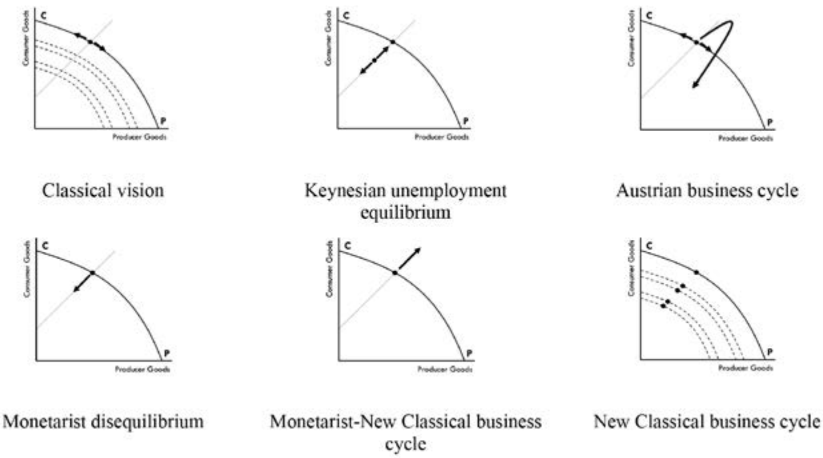
Although physical and human capital are long-lived in one sense, the subjective expectations that confer their capital character can change rapidly as entrepreneurial planners discover new information or disrupt the status quo. This creates a natural expectation of anti-persistence as entrepreneurial plans are experimentally entered into and abandoned, expectations are revised, entrepreneurs exit and enter markets, new information is received, acted on, and developed, etc. It is only when systematic monetary expansion impairs and reduces the information content of relative prices and interest rates that entrepreneurial planners' time horizons shorten and herding behavior manifests, imposing persistent long memory (Koppl 2002) in the late stages of an unsustainable expansion.

Cachanosky and Lewin's analysis of subjective valuation and production structure focuses on the need for entrepreneurial plans to be adaptable in the face of changing conditions and entrepreneurial opportunities. Awareness of changed conditions can cause entrepreneurs to rapidly change their valuation of different capital combinations (Cachanosky 2014; Cachanosky & Lewin 2014, 2016, 2018). This further implies an anti-persistent output process. Entrepreneurs select investment projects with the highest return or estimated capital value. There are always different ways to allocate the same resources (Thirlby 1952: 209–214), especially for uninvested financial capital.

Entrepreneurial experimentation normally imposes anti-persistence on output but this will be impaired if expansionary monetary policy degrades the information content of market prices, likewise imposing shorter time horizons and herding behavior on entrepreneurial planners (Lewin & Cachanosky 2018, 2019, 2020a, 2020b). This will lower the market's naturally entrepreneurial anti-persistence and may impose persistent long memory. We would expect to find the strongest evidence for this effect during the unsustainable expansion leading up to the onset of recession.

In Garrison's (2001: 248) typology of business cycle theories (Figure 4), movements parallel to the production possibilities frontier indicate (PPF) consumer and producer goods are substitutes, which is a feature of the classical business cycle model. Virtually all other business cycle models depict producer and consumer goods as complements, representing the business cycle as movements perpendicular to the PPF. Austrian business cycle theory is the only model which allows for output movements that are simultaneously perpendicular and parallel to the downward-sloping PPF, further strengthening an expectation of anti-persistence—according to Austrian business cycle theory the output choice point can and does move more frequently and potentially in more directions. Recessions, or at least the unsustainable overexpansion that precedes and causes them, should be characterized by systematic disruptions to the normal anti-persistence of entrepreneurial experimentation. Recovery restores the market's normal entrepreneurial anti-persistence.

FIGURE 4.
GARRISON'S (2001: 248) TYPOLOGY OF BUSINESS CYCLE THEORIES



The suggested causal sequence is outlined in Table 2.

TABLE 2.
SUGGESTED CAUSAL SEQUENCE FROM MONETARY EXPANSION TO IMPOSING LONG MEMORY AND INCREASING HURST EXPONENT

1. monetary expansion increases prices in certain industrial sectors which are the sites of monetary injection.
2. distorted relative prices disrupt sustainable, pre-expansionary resource allocation toward industries that initially benefit from monetary injection, and away from all others.
3. further monetary expansion drives further resource misallocation.
4. unsustainable capital structures are installed, permanently depressing output below its sustainable long-run trend even in the face of attempts to reallocate producer goods.
5. antipersitent entrepreneurial experimentation (H < ½) is suppressed raising H, and in extreme cases of unsustainability imposes persistent long-memory (H> ½) in aggregate output.

Monetary expansion is the prime suspect for imposing long memory, or at least for reducing anti-persistence, before a recession because it has an economy-wide effect in depressing interest rates. Although this is an undiscriminating, economy-wide phenomenon, it generally has stronger effects in particular areas of the economy which persistently and unsustainably overexpand. These unsustainable over-expansions are Cantillon effects (Blaug 1977) focused in particular industries, and though they can be difficult to observe ex ante, they generally become obvious after the onset of a recession. We saw the technology sector expand before the 2001 recession, and real estate, construction, and finance before the 2007-2009 Great Recession. Cantillon effects impose further persistent long memory because they are concentrated in certain sectors.

However, Minsky's (1982, 1986, 1992) Financial Instability Hypothesis (FIH) attributes recessions to the adjustment that liquidates endogenous overleveraging resulting from prolonged prosperity and expansion, where naïve agents underperceive actual risk, making the economy critically fragile before a recession. ABC and the FIH share a similar expectation of imposing herding behavior and long memory toward the end of an unsustainable expansion, which a recession washes out, reimposing the anti-persistence of entrepreneurial market process (Mulligan 2013a, 2013b, 2014; Mulligan, Lirely, & Coffee 2014). The two business cycle theories are observationally equivalent in terms of expectations for fractal long memory and anti-persistence, though the FIH views the unsustainable expansion as endogenous while ABC theory posits expansionary monetary policy as an exogenous driver. None of the alternative theories draw any connection between the entrepreneurial experimentation of market process, expansionary monetary policy, and the unsustainability of economic expansion. It might be said that ABC theory offers an additional degree of freedom for policy-induced mischief alternative business cycle theories neglect. ABC theory is also unique in recognizing that consumer and producer goods can simultaneously be both substitutes and complements, as they are in reality.

5. **Results**

Economic output may respond to external factors like money supply expansion, or it may result from endogenous, profit-seeking entrepreneurial experimentation that allocates production and productive resources, responds to price changes, substitutes among inputs and outputs, and introduces new products. Most expansion periods between recessions are long enough to compute Hurst exponents, but from 1919-2022 only a few recessions lasted long enough to allow H to be estimated by wavelets. Hs for expansion periods are given in Table 3.

TABLE 3.
HURST EXPONENTS COMPUTED BY WAVELETS
FOR BUSINESS CYCLE EXPANSIONS 1919-2022

Expansion	*expansion months*	*H*	*interpretation*
Jul 1921-Apr 1923	22	0.1430	–
Jul 1924-Sep 1926	27	0.2990	–
Nov 1927-Jul 1929	21	0.4070	–
Mar 1933-Apr 1937	50	0.4520	–
Jun 1938-Jan 1945	80	0.3090	–
Oct 1945-Oct 1948	37	0.1840	–
Oct 1949-Jun 1953	45	0.0370	–
May 1954-Jul 1957	39	0.2010	–
Apr 1958-Mar 1960	24	0.8860	+
Feb 1961-Nov 1969	106	0.6350	+
Nov 1970-Jan 1975	36	0.2480	–
Mar 1975-Dec 1979	58	0.9210	+
Nov 1982-Jun 1990	92	0.0360	-
Mar 1991-Jan 2001	120	0.1280	–
Nov 2001-Dec 2007	73	0.8360	+
Jun 2009-Jan 2020	127	0.4620	–

Note: Hs computed by wavelets. – indicates periods of anti-persistence with H < ½, + indicates periods of persistent long memory with H > ½. Hs for 12 out of 16 business cycle expansions indicate anti-persistence. Persistent long memory is found only for the 1958-1960, 1961-1969, 1975-1979, and 2001-2007 expansions, but not for any others.

Hurst exponents for the longer expansionary periods are generally less than ½, indicating anti-persistent long memory or negative serial correlation. In other words, during the expansionary periods between recessions, above-average monthly output growth is most often followed by below-average growth, and this sawtooth pattern persists for many months. This outcome is observed for twelve out of the sixteen expansions. Considered in isolation, this result does not suggest the persistent monetary expansion and unsustainable growth that is central to ABC theory, though it is possible that monetary expansion either only manifests in the late stages of an expansion—perhaps only the last two years—or that increases in the money supply during economic expansions do in fact systematically raise H, though not above ½—in other words, expansionary monetary policy may reduce the economy's normal anti-persistence in output growth, but not eliminate it completely. Anti-persistence in 12/16 expansions indicates the high level of aggregation in the IIP does not keep H from clearly indicating entrepreneurial experimentation of market process.

However, there are also a few exceptions. Hurst exponents for the brief 1958-1960 expansion and the longer 1961-1969, 1975-1979, and 2001-2007 expansions are all greater than ½, indicating persistent long memory, periods of stronger-than-average economic growth which ultimately could not be sustained, leading to the subsequent recessions. All were periods of especially strong monetary expansion, which should be more likely to yield observable Cantillon effects (Blaug 1977). These are evident in the Hs for these expansions.

Recessions are generally shorter in duration than expansions, in most cases making it impossible to compute H. For the three recessions for which Hs could be estimated, they were invariably less than ½, indicating negative serial correlation or anti-persistent long memory, shown in Table 4. Because of their very brief duration, and more particularly because of the very brief expansion period separating them, the recessions of 1980 and 1981-1982 and the recovery separating them are treated as a single recession. Anti-persistence ($H < ½$) is observed for all three recessions, consistent with expansionary/inflationary policies being abandoned, though in fact the Federal Reserve System, like most central banks, normally expands money and credit during a recession to support recovery. It may be that during recessions the economy is less able to expand systematically in response to expansionary policy stimulus compared to the preceding unsustainable expansion. The same firms and entrepreneurial planners that were so eager to invest in high-cost producer goods in the low-interest environment of the unsustainable expansion are newly reluctant, cautious, and conservative. They are hesitant to invest in the seemingly more uncertain environment, even when debt deflation offers fire-sale price reductions.

TABLE 4.
HURST EXPONENTS COMPUTED BY WAVELETS FOR BUSINESS CYCLE EXPANSIONS AND RECESSIONS 1919-2022

Preceding expansion	*Expansion months*	*H*	*Interpretation*	*Recession*	*Recession months*	*H*	*Interpretation*
Nov 1927-Jul 1929	21	0.4070	–	Aug 1929-Feb 1933	43	0.0390	–
Mar 1975-Dec 1979	58	0.9210	+	*Jan 1980-Oct 1982	22	*0.3820	–
Nov 2001-Dec 2007	73	0.8360	+	Dec 2007-May 2009	18	0.4620	–

Note: – indicates periods of anti-persistence with H < ½, + indicates periods of persistent long memory with H > ½. Persistent long memory is found only for the 1958-1960, 1961-1969, 1975-1979, and 2001-2007 expansions, but not for any others (see Table 2). * indicates two brief 1980 & 1981-1982 recessions treated as a single recession. All the recessions for which Hs can be computed by wavelets indicate anti-persistence.

When the Hurst exponent for the expansion indicates persistent long memory (½ < H ≤ 1.0), the following recession lowers H to the point where it indicates anti-persistence (0 ≤ H < ½). Even for the first part of the Great Depression, though the preceding 1927-1929 expansion has H < ½ indicating long-term anti-persistence (Peters 1999), the extraordinarily long and severe 1929-1933 recession has an H much closer to zero, that is, indicating much stronger anti-persistence. This suggests that some exogenous driver like expansionary monetary policy reduced the economy's natural anti-persistence and entrepreneurial experimentation in the late 1920s. These results indicate that the recoveries from these three recessions were characterized by the reassertion of anti-persistent market process driven by entrepreneurial experimentation. In other words, these results support ABC theory. Normal resource reallocation driven by entrepreneurial experimentation are characterized by seemingly random, anti-persistent changes at one month window intervals, but adjustments manifested over two, three, and four month windows are less frequent (Lee et al 1998).

To examine each recession we compare the period leading up to the collapse with the period following it. Whole-expansion Hurst exponents are given in Tables 3 & 4, but they may be less informative because the monetary expansion and unsustainability that precedes a recession may not manifest from the start of the expansion. Thus, we focus on the two years immediately preceding the onset of each recession, the two years centered on the onset, and the two years following. In most cases the NBER business cycle data committee considers the recession officially over before two years have elapsed, but it generally takes at least that long for labor markets to restore pre-recession levels of employment. For each recession, a time series of uniform length was constructed including the 24 months prior to, and the 24 months following and including, the first month of the recession. Each 48-month sample was also further truncated to produce three 24-month periods for each recession: 24 months before, centered on (i.e., 12 months before and after), and following, the start of each recession. Hurst exponents for each of these periods are given in Table 5.

The Hurst exponents for various 48- and 24-month periods associated with each recession are interpreted as follows. The two years preceding each recession (column a) display anti-persistence ($H < ½$) 13/17 times. The two years following each recession (column b) show anti-persistence 12/18 times. Four-year periods centered on the onset of each recession (column c) show anti-persistence 17/18 times. Two-year periods centered on the onset of recession (column d) show anti-persistence only 12/18 times. Taken together, the Hs in columns a, b, and d can be viewed as portraying a sequence transitioning from expansion through recession, where late-stage expansion Hs indicate anti-persistence 76 percent of the time, through the transition from expansion to recession, to the recovery period after the recession, where Hs both indicate anti-persistence less frequently, 67 percent of the time. In other words, the onset of a recession makes continued anti-persistence somewhat less likely. Although the Hs in column c exhibit significantly more anti-persistence, indicating that the recessionary correction resets the market's fractal character and recovery is led by risk-taking entrepreneurial planners, this is only mild confirmation of ABC.

TABLE 5.
HURST EXPONENTS, VARIOUS RECESSIONS, VARIOUS PERIODS

	Recession	*a) 24 mo pre-recession Hs*		*b) 24 mo post-recession Hs*		*c) 24 mo pre & post*		*d) 12 mo pre & post*	
1	Jan 1920- Jun 1921	Feb 1919- Dec 1920	n/a	Jan 1920- Dec 1921	0.047	Feb 1919- Dec 1921	0.256	Feb 1919- Dec 1920	0.252
2	May 1923- Jun 1924	May 1921- Apr 1921	0.338	May 1923- Apr 1925	0.072	May 1921- Apr 1925	0.050	May 1922- Apr 1924	-0.037
3	Oct 1926- Oct 1927	Oct 1924- Sep 1926	0.263	Oct 1926- Sep 1928	0.201	Oct 1924- Sep 1928	0.096	Oct 1925- Sep 1927	-1.274
4	Aug 1929- Feb 1933	Aug 1927- Jul 1929	0.444	Aug 1929- July 1931	0.388	Aug 1927- July 1931	0.419	Aug 1928- Jul 1930	0.334
5	May 1937- May 1938	May 1935- Apr 1937	0.521	May 1937- Apr 1939	0.228	May 1935- Apr 1939	0.325	May 1936- Apr 1938	0.508
6	Feb 1945- Sep 1945	Feb 1943- Jan 1945	0.650	Feb 1945- Jan 1947	0.169	Feb 1943- Jan 1947	0.385	Feb 1944- Jan 1946	0.187
7	Nov 1948- Sep 1949	Nov 1946- Oct 1948	0.224	Nov 1948- Oct 1950	0.457	Nov 1946- Oct 1950	0.014	Nov 1947- Oct 1949	0.062
8	Jul 1953- Apr 1954	Jul 1951- Jun 1953	0.115	Jul 1953- Jun 1955	0.518	Jul 1951- Jun 1955	0.004	Jul 1952- Jun 1954	-0.792
9	Sep 1957- Mar 1958	Sep 1955- Aug 1957	0.241	Sep 1957- Aug 1959	0.245	Sep 1955- Aug 1959	0.793	Sep 1956- Aug 1958	0.864
10	Apr 1960- Jan 1961	Apr 1958- Mar 1960	-0.180	Apr 1960- Mar 1962	0.248	Apr 1958- Mar 1962	0.298	Apr 1959- Mar 1961	0.126
11	Dec 1969- Oct 1970	Dec 1967- Nov 1969	0.272	Dec 1969- Nov 1971	0.190	Dec 1967- Nov 1971	0.200	Dec 1968- Nov 1970	0.178

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	Recession	*a) 24 mo pre-recession Hs*		*b) 24 mo post-recession Hs*		*c) 24 mo pre & post*		*d) 12 mo pre & post*	
12	Nov 1973-Feb 1975	Nov 1971-Oct 1973	0.281	Nov 1973-Oct 1975	0.378	Nov 1971-Oct 1975	0.362	Nov 1972-Oct 1974	0.646
13	Jan 1980-Jun 1980	Jan 1978-Dec 1979	-0.582	Jan 1980-Dec 1981	0.014	Jan 1978-Dec 1981	0.306	Jan 1979-Dec 1980	0.801
14	Jul 1981-Oct 1982	Jul 1979-Jun 1981	0.033	Jul 1981-Jun 1983	0.545	Jul 1979-Jun 1983	0.078	Jul 1980-Jun 1982	0.605
15	Jul 1990-Feb 1991	Jul 1988-Jun 1990	0.378	Jul 1990-Jun 1992	0.822	Jul 1988-Jun 1992	0.496	Jul 1989-Jun 1991	0.648
16	Mar 2001-Oct 2001	Mar 1999-Feb 2001	0.757	Mar 2001-Feb 2003	0.855	Mar 1999-Feb 2003	0.007	Mar 2000-Feb 2002	0.241
17	Dec 2007-May 2009	Dec 2005-Nov 2008	0.265	Dec 2007-Nov 2009	0.572	Dec 2005-Nov 2009	0.260	Dec 2006-Nov 2008	0.336
18	Feb 2020-Mar 2020	Feb 2018-Jan 2020	0.942	Feb 2020-Jan 2022	0.998	Feb 2018-Jan 2022	0.324	Feb 2019-Jan 2021	0.354
		Anti-persistent/ persistent	13/4 (76%)		12/6 (67%)		17/1 (94%)		12/6 (67%)

Note: Negative Hs are spurious and may simply be an artifact of the short time series lengths. Negative values should be interpreted as indicating anti-persistence ($0 \leq H < ½$). Column a shows 13/17 Hs indicating anti-persistence. Column b shows 12/18. Column c shows 17/18. Column d shows 12/18.

It is also informative to compare columns c and d, which contrast the four-year periods centered on the onset of each recession with more-focused two-year periods. The four-year periods (column c) show anti-persistence 94 percent of the time, compared with the two-year periods (column d) only 67 percent. Taken together, this presents further evidence that some exogenous driver such as expansionary monetary policy imposes persistent long memory or at least lessens the economy's normal anti-persistence which is indicative of entrepreneurial experimentation.

Table 6 examines how Hs change over the onset of a recession. When a recession starts, long memory that was manifested in the last, least sustainable stages of the expansion appears to be washed out, typically reasserting anti-persistence. This is seen in the increasingly large negative values for the (before – after), (before – centered), and (centered – after) differences in H. Furthermore, average negative values for these differences, reported as sum(H) become more negative as the observation samples pass the onset of recession, moving from left to right in Table 5. Both the number and percentage of negative, anti-persistent Hurst exponents increase systematically as the recession passes and the normal anti-persistence of entrepreneurial experimentation is restored. This confirms the ABC account of how an unsustainable expansion leads to recession, and how the recession or *market correction* reestablishes an environment where entrepreneurial experimentation can flourish.

TABLE 6.
DIFFERENCES IN HURST EXPONENTS, VARIOUS RECESSIONS, BEFORE V. AFTER V. CENTERED ON ONSET

	Recession	*Before – Centered (4a – 4c)*	*Centered – After (4c – 4b)*	*Before – After (4a – 4b)*
1	Jan 1920-Jun 1921	n/a	0.205	n/a
2	May 1923-Jun 1924	0.338	-0.072	0.266
3	Oct 1926-Oct 1927	0.263	-0.201	0.062
4	Aug 1929-Feb 1933	0.110	-0.054	0.056
5	May 1937-May 1938	0.013	0.280	0.293
6	Feb 1945-Sep 1945	0.463	0.018	0.481
7	Nov 1948-Sep 1949	0.162	-0.395	-0.233
8	Jul 1953-Apr 1954	0.115	-0.518	-0.403
9	Sep 1957-Mar 1958	-0.623	0.619	-0.004
10	Apr 1960-Jan 1961	-0.126	-0.122	-0.248
11	Dec 1969-Oct 1970	0.094	-0.012	0.082
12	Nov 1973-Feb 1975	-0.365	0.268	-0.097
13	Jan 1980-Jun 1980	-0.801	0.787	-0.014

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	Recession	*Before – Centered (4a – 4c)*	*Centered – After (4c – 4b)*	*Before – After (4a – 4b)*
14	Jul 1981-Oct 1982	-0.572	0.060	-0.512
15	Jul 1990-Feb 1991	-0.270	-0.174	-0.444
16	Mar 2001-Oct 2001	0.516	-0.614	-0.098
17	Dec 2007-May 2009	-0.071	-0.236	-0.307
18	Feb 2020-Mar 2020	0.588	-0.644	-0.056
Sum(ΔH)		-0.166	-0.805	-1.176
Fraction increasing anti-persistence		7/17 < 0	11/18 < 0	11/17 < 0
Percent increasing anti-persistence		41%	61%	65%

Note: Cumulative sum(ΔH)s indicate that the onset of recession increase the extent and frequency of anti-persistence. Fractions and percentages of negative differences, indicating increases in anti-persistence or decreases of persistent long memory, become stronger as the onset passes.

Hurst exponents associated with each recession were computed on a variety of alternative samples: 24 months prior to the onset of a recession, 24 months following the onset, 24 months prior to and following the onset, and 12 months prior to and following the onset. One question that naturally arises is which Hs are most informative? From Figure 3 we can see that Hs computed from 24 months following onset samples are more consistent, particularly noting that none are negative. Both sets of Hs rise markedly during the end of the sample in the post-Bretton-Woods era, suggesting persistent long memory (H > ½) consistent with persistent monetary expansion.

Figure 4 compares Hurst exponents computed with four-year and two-year IIP samples centered on the onset of each recession. Interestingly, although the shorter-sampled Hs might be expected to be more focused on the onset event and/or its proximate cause, the longer-sampled Hs are more consistent and never negative. Both H estimates move in tandem and also rise toward the end of the sample with the abandonment of the last link of the dollar to gold, though this effect cannot be observed after 1995.

FIGURE 3.
COMPARISON OF H COMPUTED FROM 24 MONTH
PRE- & POST-RECESSION IIP

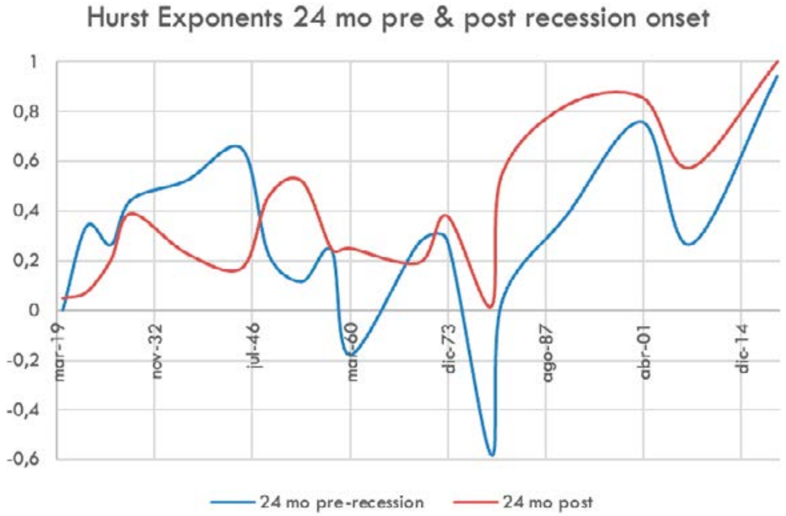
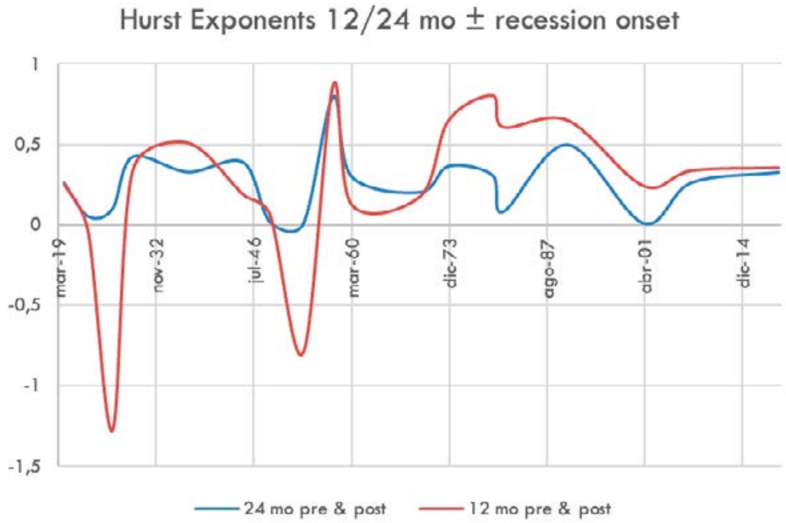


FIGURE 4.
COMPARISON OF H COMPUTED FROM 12 & 24 MONTH
PRE- & POST-RECESSION IIP



Considering all eighteen recessions experienced over the 1920-2020 period, we note first that the Hs computed from each sample before, after, and during the onset of each recession are not generally highly correlated with each other. The strongest correlations are between 12 and 24 month pre- and post-recession-onset Hs (0.5897) and between 24 month pre-onset and 24 month post-onset Hs (0.5675). Some correlations are low but negative, indicating that different ways of computing H result in statistics that move in different directions during a recession and will give very different indications, though this effect is weak.

TABLE 7.
CORRELATION MATRIX AMONG IIP RECESSION
HS ESTIMATED FOR VARIOUS SAMPLES 1920-2020

	24 mo pre-recession	*24 mo post*	*24 mo pre & post*	*12 mo pre & post*
24 mo pre-recession	1			
24 mo post	0.5675	1		
24 mo pre & post	0.0404	-0.0835	1	
12 mo pre & post	-0.0594	0.0912	0.5897	1

Note: n = 18, one H estimated for each recession from 1920-2020 over various samples as indicated.

How well do computed Hs track established business cycle indicators? This effect seems to vary substantially over time. For example, the growth rate of commercial and industrial lending, an indicator of credit expansion, has a 64% correlation with the four-year IIP H centered on the onset of recession over 1980-2020 (Table 8), but a 54% correlation over 1990-2020 (Table 9).

In the 1990-2020 sample, correlation between recession H and M2 growth is particularly strong for 24 month pre-onset H (64%) and 24 month post-onset H (90%) (Table 9), and the correlation between Hs and the interest spread are lower in absolute value and generally negative, which is expected. The interest spread used was the 10-year minus 2-year Treasury constant maturity spread. Higher M2 growth lowers the interest spread by lowering short term interest and this seems to have boosted recession Hs consistently over 1990-2020.

TABLE 8.
CORRELATIONS BETWEEN H, C&I LENDING, & INTEREST SPREAD 1980-2020

	24 mo pre-recession	*24 mo post*	*24 mo pre & post*	*12 mo pre & post*	*C&I lending growth*	*interest spread*
24 mo pre-recession	1					
24 mo post	0.9691	1				
24 mo pre & post	-0.0999	-0.0017	1			
12 mo pre & post	-0.8222	-0.7015	0.4412	1		
C&I lending growth	0.1005	0.0988	0.6420	0.3268	1	
Interest spread	0.1589	0.1674	0.0913	-0.2213	-0.1684	1

Note: n = 6.

TABLE 9.
CORRELATIONS BETWEEN H, C&I LENDING GROWTH, M2 GROWTH, & INTEREST SPREAD 1990-2020

	24 mo pre-recession	*24 mo post*	*24 mo pre & post*	*12 mo pre & post*	*C&I lending growth*	*M2 growth*	*interest spread*
24 mo pre-recession	1						
24 mo post	0.8805	1					
24 mo pre & post	-0.3603	0.0315	1				
12 mo pre & post	-0.4453	0.0261	0.8978	1			
C&I lending growth	0.5686	0.8484	0.5483	0.4512	1		
M2 growth	0.6363	0.8976	0.4651	0.3903	0.9947	1	
interest spread	-0.5271	-0.4133	-0.2149	0.1990	-0.5352	-0.5103	1

Note: n = 4.

The correlations between M2 growth, C&I lending growth, and the interest spread with H over the six 1980-2020 recessions suggest H is a strong indicator of the credit expansion that drives unsustainable expansion and causes recession.

6. Conclusion

Mulligan (2014) found strong evidence of anti-persistent long memory in resource prices ($0 \leq H < ½$), so the result found here for output is entirely consistent. Examining the 1919-2022 U.S. index of industrial production for multifractal long memory over business cycle expansions and recessions shows that recessions occur when the market's normal anti-persistence and entrepreneurial innovation is swamped by some exogenous factor like expansionary policy that imposes persistent long memory. This seems to prime the economy for a correction that restores its naturally resilient anti-persistent processes of independent, uncorrelated, *competitive* entrepreneurial experimentation.

Wavelet analysis computes Hurst exponents for most expansion periods and for a few of the longest-lasting recessions. We examined whether industrial production and general economic output are predominantly driven by organic changes in supply and demand and the random entrepreneurial innovations responding to them (anti-persistent long memory) or by unsustainable Cantillon effects of systematic monetary expansion (persistent long memory). Hurst analysis indicates that the multifractal character of industrial production varies systematically during recessions and expansions, and that recessions invariably disrupt the economy's natural entrepreneurially-driven anti-persistence. It is especially remarkable that anti-persistence can be observed so strongly and consistently in the highly-aggregated IIP series and at one-month intervals. Recovery only comes as the entrepreneurial planners who drive market process restore the economy's resilience and innovation.

Conflict of interest

The author declares that he has no conflict of interest.

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