CROVELLI ON PROBABILITY: A CRITIQUE*

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I
THE "CORRECT" DEFINITION OF PROBABILITY – A VERBAL DISPUTE

According to that old adage, if you are going to attack the king, you had better kill him. Mises, of course, is our emperor. Crovelli (2010) has launched a denunciation of him. In our view, he has not at all succeeded. The monarch, of course, cannot respond, but we, his courtiers, can. In this paper we will attempt to refute the former in defense of the latter.

Crovelli, more than once, upbraids Mises for not defining probability; for using the concepts of case and class probability, without ever explicating what these two branches have in common. And, this is a legitimate, although somewhat minor, criticism of Ludwig von Mises. In fact we observe that "probability" is essentially mathematical in meaning, whether we consult Wolfram MathWorld which states:

> "Probability is the branch of mathematics that studies the possible outcomes of given events together with the outcomes' relative

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> likelihoods and distributions. In common usage, the word "probability" is used to mean the chance that a particular event (or set of events) will occur expressed on a linear scale from 0 (impossibility) to 1 (certainty), also expressed as a percentage between 0 and 100%. The analysis of events governed by probability is called statistics.
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> There are several competing interpretations of the actual "meaning" of probabilities. Frequentists view probability simply as a measure of the frequency of outcomes (the more conventional interpretation), while Bayesians treat probability more subjectively as a statistical procedure that endeavors to estimate parameters of an underlying distribution based on the observed distribution",[1]

or the OED:

> "probability, *n.* **3.** ***Mathematics.*** As a measurable quantity: the extent to which a particular event is likely to occur, or a particular situation be the case, as measured by the relative frequency of occurrence of events of the same kind in the whole course of experience, and expressed by a number between 0 and 1.
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> An event that cannot happen has probability 0; one that is certain to happen has probability 1. Probability is commonly estimated by the ratio of the number of successful cases to the total number of possible cases, derived mathematically using known properties of the distribution of events, or estimated logically by inferential or inductive reasoning (when mathematical concepts may be inapplicable or insufficient)."

Mises defines/explains what he means by class probability and case probability, but granted that MathWorld didn't exist at the time he wrote, how difficult would it have been for him to provide a standard, authoritative definition of probability? Moreover, the dichotomy between class and case probability strictly coincides with the Knightian (1921) distinction between risk and uncertainty, which preceded Mises' terminology by decades. Ludwig von Mises maintains (thereby siding with his brother, Richard von Mises (…)) that it is only *class probability* that yields itself to numerical expression, whereas case probability does not. If we take the

[1] http://mathworld.wolfram.com/Probability.html

position that "probability" may (also) be manifested in non-mathematical ways, it does not take much to realize that this generic concept can be divided into two sub-species: the deficiency in our knowledge about statements can have a numerical expression or not, therefore, we end up with numerical probability (class probability) and non-numerical probability (case-probability); i.e., what Knight termed risk and uncertainty, respectively. How much simpler and less confusing had Mises but used Knight's terminology.

To illustrate the above, let us consider the following: what is the probability that Crovelli will write and publish a response to our rejoinder? We are inclined to say (following Richard von Mises's insight) that this question is meaningless; that is, the probability of the above statement cannot be given any numerical value.[2] And now again, we can almost hear Crovelli reply: "Aha, they beg the question! The correct definition of probability is the very point at issue and Wysocki-Block assume away the possibility that probabilities of singular events can be judged numerically." But this criticism is wrong. We are not merely assuming anything away. We now ask what number can possibly be placed on the probability that Crovelli will react to us, and defy him to comply with this request.[3] Obviously, he cannot do any such thing.

What matters to us is that when it comes to risk, assigning numerical values to a chance of the occurrence of an attribute in question is valid, whereas in the case of singular events it is not, and cannot be. We thus preserve the traditional distinction between risk and uncertainty.

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[2] This statement would not even be false if there were an infinite number of alternative universes (DeWitt and Graham, 1973; Everett, 1957; Howell, 2018; Moskowitz, 2011; Tipler, 2005, 2012, 2014), and in some of them Crovelli did respond to our present essay, and in some of them he did not. For in each of them it would be entirely up to his freely made decision, his free will (Block, 2015; Van Schoelandt, 2016), to determine whether or not he did so. We could still not generate any meaningful probabilistic number. Unlike a coin flip, or a throw of the dice, all of these "observations" would not be anything like identical.

[3] We sympathetically interpret Crovelli's claim about the possibility of assigning probabilities to singular events. To do so, we resort to the so-called betting quotient. However, this author fails to offer any principle for assigning probabilities to singular events whatsoever.

Let us press this point even harder so that we can satisfy our adversary. What about defining probability as uncertainty as such. This generic category (*genus proximum*) would then split into two subspecies: risk and mere uncertainties. And what, in turn, are the *differentia specificae* of each of them? We suggest that the former can be assigned a numerical value, whereas the latter cannot. What is more, we submit that when it comes to the former, probability would attach to a propositional function[4], while the latter lies somewhere between absolute certainty and absolute impossibility, which effectively means that any attempt at a numerical assessment would come to naught. Do we beg the question here? As long as we adhere to the common parlance (insofar as technical economics is concerned), the burden of proof remains with Crovelli.

So, what is this author's substantive criticism of Mises? Is it that since Mises is a subjectivist on some, many issues, he is logically inconsistent in not adopting this stance across the board, to wit, to probability? But why ever just because a person adopts a certain analytic tool to address one problem, must he utilize it for cases in which it patently does not apply? If all you have is a hammer, then every challenge starts to look like a nail, at least for Crovelli.

This is evidenced by the following quote:

> "The conclusion that must be drawn from this, therefore, is that the brothers von Mises are right to say that numerical probabilities cannot legitimately be applied to singular cases, if numerical probability is defined virtually synonymously with the relative frequency method. They are wrong to make this claim, however, if probability is defined as a subjective measure of man's uncertainty" (Crovelli, M 2010: 12).

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[4] For example, "the probability of a car accident caused by the inhabitants of Colorado in 2018" should be understood as the probability of "x causes a car accident in 2018", where x is an inhabitant of Colorado. This in turn would reduce to the ratio of those x's which render the propositional function true to the whole range of the variable. Since there are about 5,6 million people in Colorado, the probability of the above propositional function is the ratio of the people from Colorado that actually caused an accident 2018 to the overall number of people from that state. For the analysis of probability as predicated upon propositional functions (and not propositions) see: Mackie (1973, pp. 187–88).

Let us now, *arguendo*, take Crovelli's (2010: 12) definition of probability for granted. Our adversary contends that "probability is defined as a subjective measure of man's uncertainty". Now let us see what follows if we take Crovelli's above-cited definition to its logical extreme. Because subjective measures in and of themselves may not involve cardinal numbers, the probability calculus may never use cardinal numbers either. But now *modus tollens* kicks in: of course probability calculus resorts to cardinal numbers; therefore, probability should not be identified with a "subjective measure of man's uncertainty".

One concession is due at this point. We are always willing and able to grant the point to Crovelli: yes, there are Knightian uncertainties/Misesean Case[5] probabilities (Crovelli's singular events) that cannot be given any numerical value.[6] On the other hand, our substantive (and not definitional!) claim is that there are some Knightian risks and Mises class probabilities that matter *precisely because* they can be assigned numerical probabilities.[7]

This scholar also errs when he claims (2010: 7) "that the very existence of probability is predicated on the existence of uncertainty." Close, but no cigar. Take the case of dice or card games. We know all about them, there is no uncertainty about their results at all, at least not a as a central tendency as moments of a probability distribution or moment generating function. We know with great exactitude, for example, all there is to be known about the odds of pulling a straight flush, or rolling a snake eyes. We know rather

[5] The invocation of Knightian uncertainty and Misesian case probability; or of Knigtian risks and Misesian class probability may look confusing but the we regard the terms used in the first pair as synonymous with each other and the same applies to the second pair. Both Knight and Mises resorted to natural language to express their ideas and in natural languages, alas, two distinct words may have the same denotation.

[6] The only hope of assigning probabilities to them is to invoke the betting quotient, which Crovelli fails to do. We in turn provide it in the forthcoming section believing it is the only possible interpretation that Crovelli's subjective probabilities can be given so as to count as rational and to obey the standards of probability calculus.

[7] We conceive of probability qualifications as applying in general to statements about which we are uncertain. We posit that this is uncontroversial enough such that Crovelli is barred from the counterclaim that what uncertainty is is not yet established and we thus beg the question by invoking the term *uncertainty* in the first place.

less about any given hand, or roll of the dice, except for the long-run probability of their occurring in the case of large numbers. These phenomena are all objective. In the event of large numbers, we expect the odds of any one die[8] appearing as 16.66%. The same cannot be said about the chance of Trump winning the election for president in 2020. There will not be an indefinitely large number of such voting and even if there were, we would still be a ship in the night with no compass or rudder. We could of course guess, and even make educated guesses based on polling, past experiences, etc. But empirical generalizations such as in games of chance are beyond us. But that does not mean that this type of probability is "subjective" either. Rather, voting patterns are objective. Will people pull this lever or that one. As far as we disinterested observers are concerned, this are objective facts, to be discerned, but only after the election in late 2019.

II
NECESSARY STRUCTURAL CONSTRAINTS ON SUBJECTIVE INTERPRETATION OF PROBABILITY – PROBABILITY *AXIOMS* AS *NORMS* OF RATIONALITY

What the word *probability* must necessarily connote to be worthy of its name is some sort of numerical expression; or else, our account would be rendered totally fallacious (and uninteresting at best).[9] Salmon (1966, 64) – among others – invokes the criterion of admissibility, which reads as follows:

> "We say that an interpretation of a formal system is admissible if the meanings assigned to the primitive terms in the interpretation

[8] The expectation is all the more reasonable if we conclude that the coin or the die is physically and geometrically symmetric etc.

[9] Note that Misesian idea of probability allows for numerical assessment in the case of class probability. Alas, the same cannot be said about Crovelli's account. The latter promises to deliver some ground for assigning probabilities to singular events but we look in vain for any principles related thereto in the author's entire corpus. This section is aimed at charitably interpreting what Crovelli might have in mind so that at least some abstract structural properties of probability calculus can be satisfied.

> transform the formal axioms, and consequently all the theorems, into true statements. A fundamental requirement for probability concepts is to satisfy the mathematical relations specified by the calculus of probability […]."

As subjectivist accounts purport to measure a degree of belief, the main challenge posed thereto is how to measure the degrees of those mental states, which are of qualitative, rather than of quantitative, nature. In other words, even if we accept that the degree of belief can be given a numerical value, these values must obey the axioms of probability calculus (which are to be cited shortly). Now, the only technique of translating actions into subjective beliefs (strictly speaking, the subjective probabilities we attach to given outcomes) available to us is the so-called betting quotient.[10] It is said[11] to inversely reflect subjective probabilities. In fact, the relation between betting quotients and subjective probabilities is that of equivalence. For instance, assume we believe that the probability that A will occur is 1/3 and that it will not take place (event B will eventuate instead) is 2/3 (with the latter event being a complement of the former). Then, we are ready to bet at least at the odds 2:1 for the former as against the latter. In other words, we must be offered at least twice as much money when we bet on A because we find it twice as unlikely as event B. And conversely, if we are going to bet on A when offered at least twice as much (as compared to betting on B), the implied subjective probabilities are 1/3 that A will occur and 2/3 that B will happen.

The next thing to consider is how a person cherishing some belief (to a certain degree) reacts to buying or selling specific bets.[12] Let us assume that a person assigns the subjective probability .75 to proposition P. From this we can deduce that he would be ready to buy a bet promising $1 in case the proposition is true and $0 otherwise, for *no more* than $.75. In particular, if this individual buys

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[10] https://plato.stanford.edu/entries/dutch-book/

[11] On the idea that it is our beliefs about probabilities attached to some outcomes that guide our actions and that, therefore, our actions can be used to (retrospectively) deduce the (subjective) probabilities, see, for instance: de Finetti (2008) and Ramsey (1926).

[12] The idea that beliefs guide our actions is exploited by de Finetti (2008) and this allows him to infer probabilities (the degrees of cherished beliefs) from actions (here, accepting or rejecting particular bets).

at the price of $ .75 exactly, his expected value equals zero since the price he pays for the wager is the same as the expected value of the bet given his belief ($1× .75 = $ .75) What is more, this person would also sell this wager (that is, he would bet on not-P instead of on P) for no more than $ .25. Generally speaking, if the man assigns a probability of x (with x being a percentage) to proposition P, he is ready to pay no more than $x for the wager promising $1 when P turns out to be true and promising $0 otherwise.

The above analysis could serve as a charitable perusal of Crovelli's work because this author is rather uninformative on how he would go about deducing the promised numerical (objective) probabilities to events based on his subjective assessments. After this brief introduction, let us scrutinize the relevant material from Crovelli.

Now, here is a philosophical howler. Our author maintains the coherence of "assigning numerical probabilities to singular cases". Crovelli (2010) introduces this apparent coherence as follows:

> "Since, as was just seen, the legitimacy of assigning numerical probabilities to singular cases depends inexorably on the definition of probability, it is important to determine which definition of probability fits in with the rest of Ludwig von Mises's epistemological and praxeological system" (Mises, L [1949]1988: 12).

Crovelli is here adamant about stressing the priority of the definition of probability itself, which, as noted above, places semantic issues over substantive ones – in particular, it may dismiss frequentialism out of hand without even bothering to consider whether it is a valid method of generating (we fear to say it) *probabilities* (the chances of the occurrence of a given attribute within a well-specified reference group). Strangely enough, Crovelli is tellingly mute on how these numerical probabilities assigned to singular events are to be computed. In the most likelihood, the main thrust of his paper[13] is to argue for assigning probabilities to singular events but one may look but in vain to find anything informative delivered on the subject.

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[13] He even wrote a separate essay on "The Possibilities of Assigning Probabilities to Singular Cases, Or: Probability is Subjective Too!"; (Crovelli, 2009) which also fails to deliver the promised computation method.

What is worse, there is a symptomatic footnote no. 29 (Crovelli M, 2010: 11-12) in which Crovelli plays out his hand admitting that probabilities are merely his opinions. He does this in his criticism of Hoppe (1997):

> "We are thus free to attack and dispute probabilities as useless, inaccurate or even self-contradictory, (as does professor Hoppe in his thorough demolition of the use of probability in the rational expectations model in Hoppe (1997)), but, if we are to be faithful to subjectivist definition, we *do not* have a right to condemn other men's numbers as 'not probabilities,' just because we disagree with how they are generated. Just as we have no right to condemn another man's opinion as 'not an opinion', solely because we disagree with it, so too do we lack any right to condemn his probabilities as 'not probabilities', solely because we disagree with how they are generated. This is true, quite frankly, because *probabilities are opinions*."

So here we have it: an open statement identifying probabilities with opinions. It is difficult to come up with more of a "smoking gun" than this. Yet, these "opinions" should at least obey the axioms of probability. To strengthen out argument, let us proceed assuming, *arguendo*, that probabilities are indeed mere opinions. Our challenge is this: would Crovelli conceive of these opinions as free-floating and not obeying any axioms of probability? We claim that if this author sticks to the former, he would be proved irrational and would lose money to a cunning bookie. Now let us recall three axioms of probability, as expressed by Swinburne (2002: 5-6) holding for all classes A, B and C:

1) $\Pr(A|B) \geq 0$,[14]

which basically says that given whatever type of event that occurred (B), the probability of event A is a non-negative real number.

[14] The formulas from Swinburne (2002) expressly refer to sets. However, they can be easily extended to propositions too. Actually, it is the latter interpretation, that is assigning probabilities to propositions, that Crovelli has in mind. Despite possible ambiguities, the reader can infer from the context whether it is the probability of a proposition or the probability of a given set (given some logical space) that is meant.

2) If $B \subseteq A$, $Pr(A|B) = 1$[15],

which means that if B is a subset (proper subset inclusive) of A, then the occurrence of A given B is certain. Let us consider some simple mathematical examples and illustrate the above rule for B being a proper subset of A (when B is not a proper subset then B=A, which is trivially true). Let B denote all the natural numbers divisible by 3 and A – all the natural numbers divisible by 6. Since whatever is divisible by 6 is also divisible by 3, but not vice versa (since the latter must be also divisible by 2), we conclude that the natural numbers divisible by 6 constitute a proper subset of the numbers divisible by 3. Then, once we know that, we know that for every x, where x is divisible by 6, the probability of those x's being divisible by 3 too is 1. What is more, this axiom is also given a more intuitive rendition in terms of tautologies or necessary truths.[16]; that is:

(2) if A is a tautology[17], then pr(A)=1 [Normalization], with this condition being sometimes replaced by

(2′) if A is a logical truth, then pr(A)=1

or even

(2′′) if AA is a necessary truth, then pr(A)=1

[15] In the original formulation by Kolgomorov (1950), the second axiom assumed the following expression: $P(\Omega)=1$; that is, if the event exhausts the entire logical space, it must be certain (in particular, the probability of a tautology or any necessary truth equals 1). This holds true for the probability of the sums which, when put together, exhaust the entire logical space. For example, the probability of raining *or* not raining tomorrow in Denver equals 1 for there is no third state of affairs imaginable.

[16] https://plato.stanford.edu/entries/dutch-book/

[17] This word has a different meaning in philosophy and in mathematics. In the former case, it applies to statements that are true by definition, that yield information, only, about how we choose to use nomenclature. For example, "Bachelors are unmarried men." In the latter case, tautology refers to a statement in formal (symbolic) logic which is true regardless of the interpretation of its variables (that is, it is true merely by virtue of its operators) as long as the interpretation of the same variables is identical all along the statement. To illustrate, the statement "either x or not-x" is true whatever x stands for as long as we always substitute the same phrase for x.. For instance, in one interpretation (substitution), we end up with "it will either rain or not rain", which is also tautologically true.

3) If $A \cap B \cap C = 0$, $Pr(A \cup B|C) = Pr(A|C) + Pr(B|C)$

> To put the third axiom simply, for any two *disjoint sets* of events, the probability of the sum thereof is the sum of their respective probabilities. Sticking to our elementary arithmetics, we can exemplify axiom 3 by the following truth: the probability of a given number being *either* a real *or* an imaginary[18] number is the same as the probability for this number to be real plus the probability for it to be imaginary.

Having said that, and even granting for the sake of argument that probabilities might be Crovelli-like *subjective opinions*, we claim that these opinions (probabilistically qualified) must still obey the above-stated axioms. Therefore, we doubt whether Crovelli with his 'probabilities as opinions' would be willing and able to undertake certain bets against us. Let us see first what the violation of axiom 1 consists of and how a bookie could exploit an agent guilty of committing this logical fallacy.

What if an agent's betting quotient is negative? In this case a bookie buys the bet yielding $1 if proposition P transpires to be true and $0 otherwise. Since the agent's betting quotient is negative, the bookie is able to buy the wager *for a negative price.* Now, note that the agent is obliged to pay to the bookie $1 if P is true and $0 otherwise, with non-zero expected payment to the bookie.[19] Generally speaking, the problem with negative quotients (and hence, with the violation of axiom 1) is that the bookie pays *the negative price* for the bet and then is guaranteed (with the exception stated in the footnote 22) a positive payment, which effectively renders the agent worse off by necessity. In the light of all this, would Crovelli be so bold as to claim that probabilities (and therefore betting quotients) can be negative? Hardly. Let us now see how a man violating axiom 2 would run the risk of a 'Dutch book'[20] being

[18] Note that real numbers and imaginary numbers constitute non-overlapping sets, with non-overlapping sets being an antecedent of the axiom 3.

[19] With one exception: if P is impossibility, then the agent would necessarily owe $0 to the bookie.

[20] https://plato.stanford.edu/entries/dutch-book/

made against him. Let us adduce the pertinent fragment from Swinburne (2002: 8) again:

> "So, if contrary to the axioms of the calculus, you judge that in a two-horse race (with no ties allowed) both horses have a probability of 2/3 of winning and so you bet £2 on each at 1–2. Then you will inevitably lose £1 whatever the result of the race. This kind of argument has been extended so as to apply where people do not bet in a literal sense, but where the gains and losses resulting from their actions are measured in terms of their value to the agent rather in monetary terms, so as to show that being guided by probabilities which do not conform to the constrains of the calculus will lead in various circumstances to inevitable loss."

As far as betting quotients for tautologies are concerned, let us analyze the two-fold way an agent might err when betting on necessary truths. Let us assume that an agent assigns to some necessary truth the probability of .9. Then a bookie buys the bet (for at least $.90) and now the agent bets against a necessary truth, which means that he will pay $1 to the bookie. All in all, the agent loses at most $.10 to the bookie.

Suppose an agent estimates the probability of a necessary truth as higher than 1? Then the bookie sells the wager (for at least $1) promising to pay $1 if P turns out to be true and $0 otherwise. Since P is *ex hypothesi* a necessary truth, the bookie will pay $1 to the agent but since the former sold the bet for at least $1, it is again the bookie who benefits and it is again the agent who loses. Then again, do Crovelli's probabilities as opinions allow for numerical values (if his subjective probabilities are about numerical values at all) for necessary truths other than 1? If so, we would like to engage in some bets with this author. In short, what we hopefully managed to demonstrate in this section is that even if Crovelli's subjective probabilities (measuring the degrees of his beliefs) may be quantitatively interpreted in terms of betting quotients, they *must* conform to the three axioms of probability. After all, what matters are degrees of *rational* beliefs. And any set of beliefs that guarantees a loss to an agent regardless of the outcome of a bet can by no means be labeled in any such way.

To conclude this section, let us see how the ideas put forward here apply to both Howden (2009) and Crovelli (2009b). What will come handy is the idea that betting quotients reflect implied probabilities and that the latter must obey some structural constraints; or else – one party to a certain set of bets must inevitably suffer a loss – whatever the outcome thereof.

Now, let us see how Howden (2009: 3 – 4) conceives of odd-makers setting odds for a boxing match:

> "Last, Crovelli asserts that probability can be applied to unique trials, and not confined to repeatable series as frequency probability theorists contend. For example, the fact that casinos and bookies seem to successfully profit from applying probabilities to the odds they offer on unique events (i.e., boxing matches) provides visible proof that accurate probability distributions can be calculated for singular events. However, while the appearance of odds may seem like a probability estimate of the fight's outcome, this is merely an illusion fabricated by an odds-maker. In distinction, the odds-maker only has to have an estimate of who will win and who will lose a fight. The odds established are used to entice individuals to bet against the expected winner, in hopes of pocketing more winnings. The degree of entrepreneurial forecasting will determine how well the odds-maker has persuaded betters to erroneously choose the wrong player. The oddsmaker does not partake in an exercise in probability, but rather one of entrepreneurial forecasting. To be sure, past matches may be used to aid the entrepreneur in assigning his or her expected likelihood as to which contestant will emerge victorious from a particular boxing match. However, to say that there are 2:1 odds that boxer A will out-spar boxer B does not imply that the bookie expects boxer A to be victorious two-thirds of any one fight, nor that the final outcome will be two-thirds of a victory for A, and one-third for B. For unique events, there can only be one outcome which may not rely on the established frequency distribution. Instead, the odds are entrepreneurially forecast based upon an expectation that boxer A will prevail, *and* that the greatest profit will be earned by enticing individuals to seek higher pecuniary rewards by betting for the expected loser in the fight."

Here is Crovelli's (2009B: 2 – 3) reply:

> "Mr. Howden's extremely important discussion of odds-makers in this section also deserves notice. In my paper I claimed that since bookies and casinos are able to consistently generate accurate odds for singular events and phenomena like boxing matches this is strong prima fascia evidence that non-frequentist methods for generating numerical probabilities were not 'meaningless,' as the brothers von Mises had claimed. Mr. Howden will have none of this, and he denounces such odds as 'an illusion fabricated by an odds-maker'[…]. His reason for denying that such odds are probabilities, however, is almost embarrassingly question begging. Indeed, his evidence that such odds are not probabilities amounts to nothing more than a mistaken restatement of how bookies go about generating odds […]."

However, we demur. We claim that in the light of what was said above about the necessary structural properties of reasonable beliefs (about underlying probabilities), it seems obvious that the only method bookies must resort to have a sure-fire gain is to set odds (coupled with their respective prices) so that the implied probabilities (which are inversely reflected by the set odds) add up to more than one.

Let us now illustrate how bookies (resorting to the sheer necessity of probability calculus) trump any considerations of whether they "generate accurate odds" or "entice individuals to seek higher pecuniary rewards by betting for the expected loser."[21] Now let us imagine there is a heavyweight boxing match between Mr. A and Mr. B and a bookie sets odds in the following manner

a) 1:1 – Mr. A's victory *against* his loss or a draw (the betting price is $50)
b) 2:1 – Mr. B's victory *against* his loss or a draw (the betting price is $33,3(3))
c) 3:1 – there is a draw rather than a conclusive result (the betting price is $25)[22]

[21] Obviously, the latter makes a lot of sense but the point is, in a sense, rather trivial. Everybody would bet on "the expected loser" *only if* the odds for him are higher. In another sense, however, nobody bets on an expected loser, for if they truly believe he is going to lose, they would not bet on him in the first place.

[22] It is another example of a bookie creating a Dutch book, which means that he will necessarily benefit at the expense of *an average* gambler (which also is compatible

The implied probabilities of Mr. A's victory and Mr. B's victory, and a draw between them are, respectively, 1/2, 1/3 and 1/4 which, when added together, yield more than 1, that is 13/12. Given that, we know that there is such a configuration of betting prices such that a gain is guaranteed to a bookie in the long run[23].

Let us analyze all the possible outcomes. We posit that the prices are set in such a manner that the bookie *always* pays $100 (whatever the result, returning the bet price plus paying the stake) but he collects $50 + $ 33,3(3) + $25, adding up to $108,3(3), which guarantees a sure-fire gain of $8,3(3). As demonstrated, what it takes to succeed concerns neither guessing the probabilities of these unique events correctly nor enticing individuals to bet on the expected loser (in this example: a tie, which obviously implies the highest odds but there is still *only one* individual betting on this outcome). Rather, this stems from the pure mathematics of the situation.

In conclusion, betting quotients (and hence implied probabilities) is a powerful weapon which allows bookies to make gains with confidence without resorting to any entrepreneurial skills at all. On the contrary, what it takes is the knowledge of calculus and the realization that when the probabilities of disjointed events (exhausting the logical space) add up to more than one, a bookie will certainly benefit while individuals betting on particular outcomes *only gamble* and therefore, they can either lose or gain.

III
THE STATISTICAL PROBABILITY – THE EXISTENCE OF COLLECTIVES – A MAJOR EMBARRASSMENT FOR CROVELLI (CASINOS, INSURANCE MARKET ETC.)

The other reason why Crovelli is erroneous is that statistical probability is a fact. Casinos necessarily benefit as they are acutely

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with the fact that some *particular* gamblers will be rendered better off as a result of the bet. After all, one of the outcomes necessarily occurs and there are some bets placed on it).

[23] Out of the infinitely many such prices, one set is specified above.

aware of the underlying probabilities in games of chance (the probabilities being normally arrived at via the frequentist method!). There is an obvious advantage of empirically obtained frequencies over merely assumed equiprobabilities of possible outcomes (as in the classical definition with its adherence to the principle of indifference[24]) as well as over subjective (degrees of beliefs) probabilities.

To that effect, let us quote Mackie (1973:161):

> "The classical definition, thus developed, will identify the probability of a possible event with the ratio of the number of 'favourable' possibilities to the total number of possibilities in a set of equiprobable possibilities. What is essential in this procedure is the use of the principle of indifference (or of insufficient reason) to determine equiprobability, that is, the claim that alternatives are equally probable is we have no reason to expect one rather than another. Now with some sense of 'probable', this principle would be simply false. E.g. with a limiting frequency sense: a die may be loaded, and we may not know this; if so, we have no reason to expect it to fall 6 uppermost rather than, say, 2 uppermost, and yet the limiting frequency of the former may be markedly higher than the limiting frequency of the latter."

First, once the limiting frequency value is established (for example, a dice falls 6 uppermost slightly more often than 2), this result is obviously much more informative than the mere assumption of the principle of indifference (that is, in the absence of any other information on relative frequencies, as in the classical definition of probability cited above). Second, and more crucially, let us imagine that Crovelli has just been tossing a coin for hours and has thus far obtained the .7 frequency of heads and only .3 frequency of tails. In the light of that, what should Crovelli's *reasonable belief* consist of?[25] Should he stick to the classical definition and ascertain that since all the forthcoming tosses are independent and there are

[24] The formulation of classical definition is normally attributed to Laplace (1814), whereas it was Keynes (1921) who came up with the principle of indifference intending to overcome the shortcomings (esp. the apparent circularity) of the former.

[25] We assume we are dealing with an equally weighted, or "fair" coin.

merely two possibilities, the principle of indifference[26] commits him to say that the probability of each and every next toss is still 1/2? Or should he cherish any belief now since after all "probabilities are opinions"? But this sheer ignorance[27] is unjustified since Crovelli has already obtained some information, that is the relative frequencies of heads and tails as a result of his long-run experiment.

This point is pellucidly illustrated by Mackie (1973: 201):

> "What is, at any moment, reasonable for someone to believe is relative to the information that he then has. But of course if he could easily get further relevant information, and the problem is of some importance to him, it will be sensible for him to get that information first […]. A judgement based on a greater quantity of relevant information is a better judgement. And if we are asked *how* it is better, we can say without blushing that it is more likely to be right. Admittedly, in the particular case where a penny is in fact unbiased, the man who bases his opinion as to whether it will fall heads at the next toss on the principle of indifference applied directly to the alternatives 'heads' and 'tails' will be no less likely to be right than the man who, by a long series of trials, first confirms the hypothesis that the limiting frequency for heads is ½ and then applies the principle of indifference […] But this is a particular case. If we start off not knowing whether the penny is biased or not, it is clearly reasonable to say that the man who experiment thoroughly is more likely to be right even about a single later result."

The final statement is very powerful indeed. When applied to our imaginary scenario with Crovelli running the experiment which yielded .7 frequency of heads, it directly implies that *the*

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[26] We are using this term in a very different manner than that rejected by Austrian economics. See on that:

Barnett, 2003; Block, 1980, 1999, 2003, 2007, 2009A, 2009B, Block and Barnett, 2010; Callahan, 2003; Collingwood, 1945; Hoppe, 2005; Hulsmann, 1999; Machaj, 2007; Rothbard, 2004, pp. 265, 267; Wysocki, 2016; Wysocki and Block, 2018.

[27] What we mean by that is, technically speaking, the maximum entropy; that is, out of k events, each is 1/k probable. In our case (tossing a coin), the highest entropy (which is the smallest amount of information possible) commits us to ascertain the probability of 1/2 to either event, be it either falling heads or tails. For details, see: Shannon (1948).

information about the obtained relative frequencies should have a bearing even on the reasonable degree of belief about any future tosses.[28] In other words, equipped with the principle of indifference and the above-stated relative frequencies , Crovelli (as an adherent of subjective probabilities) *should* believe that the probability of tossing heads at any future[29] single toss is .7. With the acquisition of the relative frequencies of heads and tails *the principle of indifference* basically says that if do not know anything over and above the mere fact that this particular coin exhibits a slight bias in favor of heads, our expectation as to any further tosses should be adjusted accordingly. Mackie goes further than either of the Mises brothers[30] and implies that the obtained frequencies (coupled with the principle of indifference) may serve to derive the corresponding probability of "a single later result"; that is, if all Crovelli knows is that this very coin exhibits such and such frequencies of the two attributes in question, he should reasonably believe that at any later toss, the probability of heads is .7, and that of tails is .3. Yet, this would be most unwelcome to Crovelli since it would treat frequencies as prior to beliefs.[31] We can only conclude that this author to the contrary notwithstanding it is the acquired information (knowledge) that helps to adjust our beliefs and not vice versa.

There is an empirical argument for the validity of frequentialist sort of probability, and that is mass phenomena. After all, there *are*

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[28] Certainly, we should bear in mind that Richard von Mises would find the probability of any single toss utterly meaningless.

[29] Assuming that throughout the experiment the physical properties of the die are not subject to any changes.

[30] Let us recall that neither von Mises brother would draw from the fact of class probability any numerical conclusions related to case probabilities. To that effect, Ludwig von Mises [1949] 1998, p. 107) said: "Class probability means: We know or assume to know, with regard to the problem concerned, everything about the behavior of a whole class of events or phenomena; but about the actual singular events or phenomena we know nothing but that they are elements of this class."

[31] This finding is another reasonable objection to purely subjective interpretations of probability: not only should subjective beliefs be constrained by probability axioms cited above but also prior probabilities cannot be purely arbitrary. If Crovelli tossed a coin one hundred times in a row and at each and every toss, it landed on heads, it should give him pause. It would be odd for him to *subjectively* believe that at the next toss the chance is fifty-fifty that tails will appear.

the phenomena which exhibit limiting frequency value and satisfy the property of randomness (which is understood as insensitivity to the method of place selection)[32], as specified by Richard von Mises (1981 [1957]: 24):

> "If the same method, or any other, simple or complicated, method of selection is applied to the sequence of dice casts, the effect will always be nil; the relative frequency of the double 6, for instance, will remain, in all selected partial sequences, the same as in the original one (assuming, of course, that the selected sequences are long enough to show an approach to the limiting value), This impossibility of affecting the chances of a game by a system of selection, this uselessness of all systems of gambling, is the characteristic decisive property common to all sequences of observations of mass phenomena which form the proper subject of probability calculus."

It is precisely for this reason that casinos cannot be outplayed. Even the most cunning gambler is unable to come up with the method of place selection that would change the relative frequencies (say, of an *unbiased* dice) in his favour and let him have an edge over the casino. This, as labelled by Mises (1981 [1957]: 24), "complete lawlessness" is just a brute fact. The consecutive throws of an unbiased dice may constitute a collective but we can never be sure a priori[33] whether this dice is *unbiased* until we prove that it really is by experimentally demonstrating that the throws thereof conform to the principle of randomness and exhibit limiting frequency value of 1/2 in the long run. Therefore, and more generally, if a given population approaches some limiting frequency and is random (that is insensitive to place selection)[34], then given the probability of a given attribute obtained via frequency method, we can

[32] "Place selection" is simply selecting some proper sub-set out of our original collective. Insensitivity to place selection means that the frequency of a given attribute (in the original collective) is not going to change in the target sub-class.

[33] Note that on frequentist approach, a priori probability is in principle empirically refutable.

[34] Certainly, insensitivity to all place selections would be too strong a requirement for any finite series. After all, there are such sub-series that would change the frequency of an attribute. Yet, Mises' point fully applies to infinite series.

apply to this population all the fundamental operations of probability calculus. And such populations *do* occur. It is via frequentialist method that a dice is tested for being unbiased; that is if all the outcomes approach the limiting frequency value of 1/6, only then can a dice be said to be unbiased. No a priori probability of the principle of indifference (in the absence of any prior information) would do. In his *magnum opus,* Mises (1981 [1957]) cites many populations which constitute collectives – the examples range–from games of chance to mortality rates[35] (exploited by insurance companies and analyzed by actuarial mathematics). In short, whether a population is a collective is *an empirical issue* and should be not dismissed out of hand by any stipulative definition of probability. For what matters is that some uncertainties (broadly understood) be quantifiable. If this fact is ascertained then the frequentialist definition of probability may justifiably be adopted.

Finally, let us give some more substance to our claim that some uncertainties (broadly understood) matter *since* we can *quantify* them (risks). In order to do so we quote the incisive excerpt from Knight's (1971: 212–13) *magnum opus*:

> "[…], the bursting of bottles does not introduce an uncertainty or hazard into the business of producing champagne; since in the operations of any producer a practically constant and known proportion of bottles burst, it does not especially matter even whether the proportion is large or small. The loss becomes a fixed cost. . . . And even if a single producer does not deal with a sufficiently large number of cases of the contingency in question . . . to secure constancy in its effects, the same result may easily be realized, through an organization taking in a large number of producers. This, of course, is the principle of insurance, as familiarly illustrated by the chance of fire loss. No one can say whether a particular building will burn, and most building owners do not operate on a sufficient scale to reduce the loss to constancy. . . . But as is well known, the effect of insurance is to extend this base to cover the operations of a large number of persons and convert the contingency into a fixed cost."

[35] The implicit expectation accompanying all the actuarial data is that similar frequencies will obtain in the near future. Needless to say that this expectation is not a matter of necessity but rather of contingency.

Do we really, in the light of the fact that "a practically constant and known proportion of bottles burst", need to delve into the problem of "the correct (sic!)" definition of probability when the only thing that matters here is *the discovery* of bottles-not-bursting-to-bottles-bursting ratio? This certainly is a rhetorical question. What is more, we can even concede, arguendo, the point to Crovelli that the universe is governed by time-invariant causation. But by no means does this force us to adopt his subjective definition of probability. A sufficient condition for *any* probability talk (be it class or case probability) is an element of our ignorance. In the example cited above, we are not sure which particular bottle is going to burst but it hardly matters for we know everything about the entire class of such events (that is, we know the overall proportion of bottles bursting) to turn this contingency into a fixed cost.

IV
DETERMINISM CALLS FOR SUBJECTIVE PROBABILITY – CROVELLI'S NON-SEQUITUR FALLACY

As was hinted in passing above, even the assumption of determinism does not oblige us to adopt the subjective definition of probability. We claim that this is a *non-sequitur* on Crovelli's part. That this author clings to the idea that invariable causal powers impose on us the subjective definition of probability is evidenced in what follows (2010: 8– 9):

> "If the world is deterministic, in the sense that every event has a cause of some sort, then any uncertainty man might have about what goes on in the world must be a result in man's own mental limitations. *Probability in such a world would thus necessarily be a measure of man's subjective beliefs about the world, rather than an 'objective' measure of a property that exists in the world, because all outcomes, events and phenomena in a causally deterministic world have absolutely certain causes.* If man were in a position to know in advance all of the causal factors affecting any given event or phenomenon, he would not have to resort to the round-about

> methods of probability to predict outcomes. He would know in advance, and for certain, whether any given event would or would not occur."

We tackle this argument by Crovelli on two counts. First, let us test it for the validity of its entailment. We assume for the sake of argument the antecedent; that is, the world is deterministic. We still part company with Crovelli as far as the consequent goes; to wit, we do not believe that what necessarily follows is the validity of subjective beliefs. On the other hand, Crovelli is right saying that there would be no need to employ any concept of probability if man were omniscient. Fair enough, ignorance is a necessary (and sufficient!) prerequisite for the concept of probability to be operative. Yet, it does not follow that since ignorance is obviously of an epistemological (rather than an ontological[36]) nature, probability must refer to subjective beliefs. Before we consider what aspects of reality obtained frequencies reflect, we should most empathically stress that we share Crovelli's intuition (although that is not the only coherent world-view by any means) that once *all* the physical conditions of throwing a dice are specified (its momentum, the angle at which it is thrown, the distance from the floor etc.), they *necessitate* one particular outcome. In other words, if all the values of all the variables relevant to the prediction of an outcome are known, there would be no room for probability qualifications since a given outcome would be certain. Still, we claim that by obtaining frequencies we do learn something about reality. Let us consider what we learn when the limiting frequency value for heads when tossing a coin is about 1/2. We side with Crovelli believing that if all the variables causally involved in bringing about either heads or tails were known for sure, either result would be known in advance and with absolutely certainty. Yet, the values of all these variables are not known (in particular, we are in the dark as to what is the momentum of a coin toss, what is its exact distance to the floor etc.) So, what do we learn when we obtain in the long run roughly 1/2 frequency for heads? We claim that we learn that the coin is unbiased;

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[36] After all, Crovelli (2010) argues that the world is governed by time-invariant causation.

for all the other variables are by definition randomly distributed over all the tosses in the experiment. After all, we do not control for momentum. Neither is the distance from the floor fixed. We simply do not care about all the other causal factors. What remains *constant* throughout the experiment is the dice itself! All the other factors (momentum, angle, distance etc.) are necessarily random then.[37] Therefore, this result of the experiment sheds light on reality (that is, on the geometry of a dice) and not on subjective beliefs. This clearly runs counter to Crovelli's insight.

Second of all, we believe that by no means it is necessary to assume time-invariant causation. There is substantial literature on the so-called *objective chance and propensity*.[38] To that effect, let us quote Mackie (1973: 179–180):

> "Objective chance is a counterpart of the power of necessity in causes for which Hume looked in vain. Just as the power in a cause would be something present in every instance of a certain kind of cause which somehow *guaranteed* the subsequent occurrence of the corresponding effect, so a penny's having, at each toss, a certain chance of falling heads and a certain (perhaps different) chance of falling tails would be something present in the initial stages of every individual tossing process which *tended* to produce the result 'heads' and *tended* to produce the result 'tails', where these tendencies might be either equal or unequal. A clearer example might be a four-sided top which, when it stopped spinning, would lie with one of the four sides (marked '1', '2', '3', and '4')

[37] Obviously, we make a tacit assumption after Kries (1886, p. 55), who believed that other causal factors, across their respective extensions, contribute equally to bringing about either 1,2,3,4,5 or 6 falling uppermost on a dice. To clarify, let us assume that the outcome of a dice is a function of the geometry of a dice and the momentum of its throw. The momentum may vary across some spectrum. Kries believed that when this spectrum is divided into 6 equal sub-spectra, any value within each of them, *ceteris paribus*, will bring about their respective outcomes. In other words, given a certain geometry of a dice, if sub-spectrum 1obtains (that is, the dice was thrown with the momentum ranging from 0-1), the dice will fall 1 uppermost; if sub-spectrum 2 obtains (that is, the dice was thrown with the momentum ranging from 1-2), the dice will fall 2 uppermost etc.

[38] Normally, it is Popper (1957) who is credited with the invention of propensity interpretation of probability. This author's concern was – among others – to make sense of assigning probabilities to single cases in the realm of quantum mechanics.

> uppermost. Each spin of this top might have a certain *chance-distribution*, say 0,3, for side 1, 0.26 for side 2, 0.2 for side 3, and 0.24 for side 4. That is, the tendencies in this case would be nearly but not quite equal, the tendency for the top to come to rest with side 1 up would be the strongest, with side 3 the weakest, and the others in between. (The chances in such a chance-distribution would *tend* to produce the various results, but of course would not *guarantee* any of them. It would, presumably, guarantee a distribution of *limiting* frequencies in an indefinitely extended sequence of spins corresponding exactly to the distribution of chances, provided that the top and the way of spinning it did not change."

The last sentence of this illuminating quote is crucial. The analogy to our previous analysis (with the dice being of the correct geometry and every other factor randomly varying) is crystal-clear. In the case of the spin invoked by Mackie, once we know that the world is basically governed by objective chances (propensities) and not by invariant causes, our *empirically obtained frequencies* can reflect those *objective dispositions (propensities)* in question! To guarantee this, it would be enough to control for other variables. With the values of other variables fixed, it would be a varying objective chance that would be reflected by the duly obtained frequencies. To give but one example, if the objective chances are as specified by Mackie, the relative frequencies (of the spin lying on side 1, 2, 3 or 4) obtained in an experiment controlling for all the other variables should in the long run exactly reflect the ratios of their respective objective chances. This is stipulated in Mackie's example. Concluding, we hope that he managed to show that even if the antecedent in Crovelli's reasoning is sound, the whole reasoning is still an instance of the *non-sequitur*. And finally, we cast some doubt on time-invariant causality showing that objective chances are seriously considered; and what is more, it is frequencies that can illuminate the former.

V
CONCLUSION

We are grateful to Crovelli (2010). In the lexicon of criticisms of Ludwig von Mises, his contribution stands head and shoulders over most of them, all too many of which amount to mere name-calling. In contrast, this author's condemnation was thoughtful, reasonable and well-articulated. However, we cannot see our way clear to agreeing with him regarding his major thesis.

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